WEEKLY TEST TYJ-01 MATHEMATICS SOLUTION 04 AUGUST 2019

31. (b) $\alpha + \alpha^2 = 30$ and $\alpha^3 = p$

$\alpha^2 + \alpha - 30 = 0 \Rightarrow (\alpha + 6)(\alpha - 5) = 0 \Rightarrow \alpha = -6, 5$

$\therefore p = \alpha^3 = (-6)^3 = -216$ and $p = (5)^3 = 125$

$p = 125$ and -216 .

32. (a) Let roots are α and β . Given $\alpha + \beta = -1$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{6} \Rightarrow \frac{\alpha + \beta}{\alpha\beta} = \frac{1}{6} \Rightarrow \alpha\beta = -6$

Hence the equation, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 + x - 6 = 0$

33. (c) α, β are the roots of the equation $x^2 - 3x + 1 = 0$

$\therefore \alpha + \beta = 3$ and $\alpha\beta = 1$

$S = \frac{1}{\alpha - 2} + \frac{1}{\beta - 2} = \frac{\alpha + \beta - 4}{\alpha\beta - 2(\alpha + \beta) + 4}$

$= \frac{3 - 4}{1 - 2.3 + 4} = 1$

and $P = \frac{1}{(\alpha - 2)(\beta - 2)} = \frac{1}{\alpha\beta - 2(\alpha + \beta) + 4} = -1$

Hence the equation whose roots are $\frac{1}{\alpha - 2}$ and $\frac{1}{\beta - 2}$ are $x^2 - Sx + P = 0 \Rightarrow x^2 - x - 1 = 0$.

34. (b) α, β be the roots of $ax^2 + bx + c = 0$

$\therefore \alpha + \beta = -b/a$, $\alpha\beta = c/a$

Now roots are $\alpha - 1, \beta - 1$

Their sum, $\alpha + \beta - 2 = (-b/a) - 2 = -8/2 = -4$

Their product, $(\alpha - 1)(\beta - 1) = \alpha\beta - (\alpha + \beta) + 1$

$= c/a + b/a + 1 = 1$

$\because$ New equation is $2x^2 + 8x + 2 = 0$

$\therefore b/a = 2$ *i.e.* $b = 2a$, also $c + b = 0 \Rightarrow b = -c$.

35. (b) Since roots of the equation $x^2 - 5x + 16 = 0$ are α, β .

$\Rightarrow \alpha + \beta = 5$ and $\alpha\beta = 16$ and $\alpha^2 + \beta^2 + \frac{\alpha\beta}{2} = -p$

$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta + \frac{\alpha\beta}{2} = -p \Rightarrow 25 - 32 + 8 = -p$

$\Rightarrow p = -1$ and $(\alpha^2 + \beta^2)\left(\frac{\alpha\beta}{2}\right) = q$

$\Rightarrow \left[(\alpha + \beta)^2 - 2\alpha\beta\right]\left[\frac{\alpha\beta}{2}\right] = q \Rightarrow q = [25 - 32]\frac{16}{2} = -56$

So, $p = -1, q = -56$.

36. (c) Equation $x^2+kx-24=0$ has one root is 3.

$\Rightarrow 3^2-3k-24=0 \Rightarrow k=5$

Put $x=3$ and $k=5$ in options, only (c) gives the correct answer.

37. (d) Here, $\alpha+\beta=-2$ and $\alpha\beta=4$

$\therefore \frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}$

$=\frac{(-2)^3-3(-2)(4)}{(4)^3}=\frac{16}{64}=\frac{1}{4}$.

38. (a) Given roots are $3p-2q$ and $3q-2p$.

Sum of roots = $(3p-2q)+(3q-2p)$ = $(p+q)=\frac{5}{3}$

Product of roots = $(3p-2q)(3q-2p)$

= $9pq-6q^2-6p^2+4pq$ = $13pq-2(3p^2+3q^2)$

= $13\left(\frac{-2}{3}\right)-2(5p+2+5q+2)$

= $13\left(\frac{-2}{3}\right)-2\left[5\left(\frac{5}{3}\right)+4\right]$

= $\frac{-26}{3}-2\left[\frac{25}{3}+4\right]$ = $\frac{-100}{3}$

Hence, equation is $3x^2-5x-100=0$.

39. (d) Subtracting, we get $2hx=56$ or $hx=28$

Putting in any, $x^2=49$

$\therefore \left[\frac{28}{h}\right]^2=7^2 \Rightarrow h=4 (h>0)$

40. (c) Let $y=\frac{x^2-3x+4}{x^2+3x+4}$

$\Rightarrow (y-1)x^2+3(y+1)x+4(y-1)=0$

For x is real $D\ge 0$

$\Rightarrow 9(y+1)^2-16(y-1)^2\ge 0$

$\Rightarrow -7y^2+50y-7\ge 0 \Rightarrow 7y^2-50y+7\le 0$

$\Rightarrow (y-7)(7y-1)\le 0$

Now, the product of two factors is negative if one in $-ve$ and one in $+ve$.

Case I : $(y-7)\ge 0$ and $(7y-1)\le 0$

$\Rightarrow y\ge 7$ and $y\le\frac{1}{7}$. But it is impossible

Case II : $(y-7)\le 0$ and $(7y-1)\ge 0$

$\Rightarrow y\le 7$ and $y\ge\frac{1}{7}\Rightarrow\frac{1}{7}\le y\le 7$

Hence maximum value is 7 and minimum value is $\frac{1}{7}$

41. (d) Let $y=\frac{x^2+34x-71}{x^2+2x-7}$

$\Rightarrow x^2(y-1)+2(y-17)x+(71-7y)=0$

For real values of x, its discriminant $D\ge 0$

$\Rightarrow 4(y-17)^2-4(y-1)(71-7y)\ge 0$

$\Rightarrow (y^2-3+y+289)-(71y-7y^2-71+7y)\ge 0$

$\Rightarrow y^2-14y+45\ge 0 \Rightarrow (y-5)(y-9)\ge 0$

It is possible when both $y-5$ and $y-9$ are negative or both positive. Let $y-5\le 0\Rightarrow y\le 5$ and $y-9\le 0\Rightarrow y\le 9$.

Hence $y\le 5$ .....(i)

If $y-5 \geq 0 \Rightarrow y \geq 5$ and $y-9 \geq 0 \Rightarrow y \geq 9$

Hence $y \geq 9$. .....(ii)

Therefore y does not lie between 5 and 9.

43. (d) $x^2 - 4x < 12$

$\Rightarrow x^2 - 4x - 12 < 0 \Rightarrow x^2 - 6x + 2x - 12 < 0$

$\Rightarrow (x-6)(x+2) < 0 \Rightarrow -2 < x < 6$.

44. (a) According to given condition,

$4a^2 - 4(10-3a) < 0 \Rightarrow a^2 + 3a - 10 < 0$

$\Rightarrow (a+5)(a-2) < 0 \Rightarrow -5 < a < 2$.

45. (b) Given equation is $x^3 - 3x + 2 = 0$

$\Rightarrow x^2(x-1) + x(x-1) - 2(x-1) = 0$

$\Rightarrow (x-1)(x^2 + x - 2) = 0 \Rightarrow (x-1)(x-1)(x+2) = 0$

Hence roots are $1, 1, -2$